Progression in Written Addition

Year 1 Add with numbers up to 20

Use numbered number lines to add, by counting on in ones. Encourage children to start with the **larger** number and count on.

Children should:

Have access to a wide range of counting equipment, everyday objects, number tracks and number lines, and be shown numbers in different con- texts.

Read and write the addition (+) and equals (=) signs within number sen- tences.

Interpret addition number sentences and solve missing box problems, using concrete objects and number line addition to solve them: 8 + 3 = |

15 + 4 = ... 5 + 3 + 1 = + = 6

This builds on from prior learning of adding by combining two sets of objects into one group (5 cubes and 3 cubes) in Early Years.

Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.

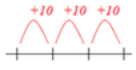
8 + 5

Year 2 Add with 2-digit numbers

Developing mental fluency with addition and place value involving 2-digit numbers, then establish more formal methods.

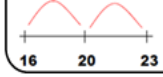
Add 2-digit numbers and tens:

27 + 30



Add 2-digit numbers and units:

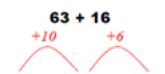
16 + 7



Use empty number lines, concrete equipment, hundred squares etc. to build confidence and fluency in mental addition skills.

Add pairs of 2-digit numbers, moving to the partitioned column method when secure adding tens and units:

63 + 16



23 + 34:

20	+ 3
+ 30	+ 4
50	+ 7
= 57	

STEP 1: Only provide examples that do NOT cross the tens boundary until they are secure with the method itself.

58 + 43:

50	+ 8
40	+ 3
90	+ 11
= 101	

STEP 2: Once children can add a multiple of ten to a 2-digit number mentally (e.g. 80+11), they are ready for adding pairs of 2-digit numbers that DO cross the tens boundary (e.g. 58 + 43).

STEP 3: Children who are confident and accurate with this stage should move onto the expanded addition methods with 2 and 3-digit numbers. (see Y3).

To support understanding, pupils may physically make and carry out the calculation with Dienes Base 10 apparatus or place value counters, then compare their practical version to the written form, to help them to build an understanding of it.

Addition

Year 3 Add numbers with up to 3-digits

Introduce the **expanded column addition** method:

200	+ 30 + 6
+ 100	+ 40 + 3
300	+ 70 + 9
= 309	

Add the **units/ones** first, in preparation for the compact method.

In order to carry out this method of addition:

Children need to recognise the value of the hundreds, tens and units without recording the partitioning.

Pupils need to be able to add in columns.

Move to the compact **column addition** method, with "exchanging":

236	
+ 73	
309	
1	

Add **units** first.

Exchange" numbers **underneath** the bottom line.

Children who are very secure and confident with 3-digit expanded column addition should be moved onto the **compact column addition** method, being introduced to exchanging" for the first time. Compare the expanded method to the compact column method to develop an understanding of the process and the reduced number of steps involved.

Remind pupils the actual value is **three tens** add **seven tens**, not three add seven", which equals **ten** tens.

Year 4 Add numbers with up to 4 digits

Move from expanded addition to the compact column method, **adding units first** and "exchanging" numbers **underneath** the calculation. Also include money and measures contexts.

e.g. 3517 + 396 = 3913

3517	
+ 396	
3913	
1 1	

Introduce the **compact column addition** method by asking children to add the two given numbers to- gether using the method that they are familiar with (expanded column addition— see Y3). Teacher models the compact method with exchanging, asking children to discuss similarities and differences and establish how it is carried out.

Add **units** first.

Exchange" numbers **underneath** the bottom line.

Reinforce correct place value by reminding them the actual value is 5 hundreds add 3 hun- dreds, **not** 5 **add** 3, for example.

Use and apply this method to money and measurement values.

Year 5 Add numbers with more than 4 digits

including money, measures and decimals with different numbers of decimal places.

£23.59	
+ £7.55	
£31.14	

The decimal point should be aligned in the same way as the other place value columns, and must be in the same column in the answer.

23,481	
+ 1362	
24843	

Numbers should exceed 4 digits.

19.01	
3.65	
+ 0.70	
23.36	

Pupils should be able to add more than two values, carefully aligning place value columns.

Say "6 tenths add 7 tenths" to reinforce place value.

Empty decimal places can be filled with zero to show the place value in each column.

Children should:

Understand the place value of **tenths and hundredths** and use this to align numbers with different numbers of decimal places.

Year 6 Add several numbers of increasing complexity

23.361	
9.08	
59.77	
+ 1.38	
93.511	
2 1 2	

Adding several numbers with different numbers of decimal places (including money and measures):

Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.

Zeros could be added into any empty decimal places, to show there is no value to add.

Empty decimal places can be filled with zero to show the place value in each column.

81,059	
3,668	
15,301	
+ 20,551	
120,579	
1 1 1 1	

~~Adding several numbers with more than 4 digits.~~

Progression in Written Subtraction

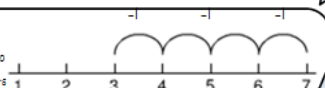
Year 1 Subtract from numbers up to 20

Children consolidate understanding of subtraction practically, showing subtraction on bead strings, using cubes etc. and in familiar contexts, and are introduced to more formal recording using number lines as below:

Read, write and interpret number sentences with – and = signs.

Subtract by taking away

Count back in ones on a numbered number line to take away, with numbers up to 20:

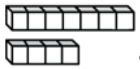


7 – 4 = 3

Model subtraction using hundred squares and numbered number lines/tracks and practically.

Find the „distance between"

This will be introduced practically with the language **find the distance between** and **how many more?** in a range of familiar contexts.



Seven is 3 more than four"

„I am 2 years older than my sister"

Mental subtraction

Children should start recalling subtraction facts up to **and within** 10 and 20, and should be able to subtract zero.

Subtraction

Year 2 Subtract with 2-digit numbers

Subtract on a number line by counting back, aiming to develop mental subtraction skills.

Use Dienes blocks for subtraction calculations too.

This strategy will be used for:

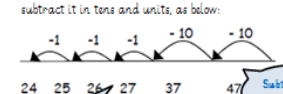
2-digit numbers subtract units (by taking away / counting back) e.g. 36—7

2-digit numbers subtract tens (by taking away / counting back) e.g. 48—30

Subtracting pairs of 2-digit numbers (see below)

Subtracting pairs of 2-digit numbers on a number line:

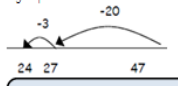
47 – 23 = 24 Partition the second number and subtract it in tens and units, as below:



Then subtract units.

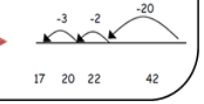
Subtract tens first.

Move towards more efficient jumps back, as below:



Combine methods with use of a hundred squares to reinforce understanding of number value and size.

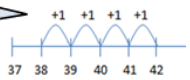
Teaching children to **bridge through ten** can help them to become more efficient, for example 42—25:



Mental strategy – subtract numbers close together by **counting on:**

Start with the smaller number and count on to the largest.

42 – 38 = 4



Many mental strategies are taught. Children are taught to recognise that when numbers are close together, it is more efficient to **count on** the difference. They need to be clear about the relationship between addition and subtraction.

Subtraction

Year 3 Subtracting with 2 and 3-digit numbers.

Introduce **partitioned column subtraction** method.

STEP 1: introduce this method with examples where **no exchanging** is required.

84 – 35 = 54

80 + 9

– 30 + 5

50 + 4

When learning to exchange", explore partitioning in different ways" so that pupils understand that when you exchange, the **VALUE** is the same (e.g. 72 = 70+2 = 60+12 = 50+22 etc. Emphasise that the **value hasn't changed** we have just partitioned it in a different way.

STEP 2: introduce exchanges" through practical subtraction. Make the larger number with Base 10, then subtract 47 from it.

72 - 47

600 + 12

– 40 + 7

20 + 5 - 25

Before subtracting 7" from the 72 blocks, they will need to exchange a row of 10 for ten units. Then subtract 7, and subtract 4 tens.

STEP 3: Once pupils are secure with the understanding of exchanges", they can use this partitioned column method to subtract any 2 and 3-digit numbers.

238 - 146 = 92

100

200 + 30 + 8

– 100 + 40 + 6

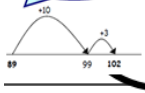
0 + 90 + 2

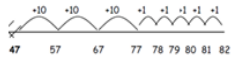
Subtracting money: partition into e.g. £1 + 30p + 8p

Counting on as a mental strategy for subtraction:

Continue to reinforce counting on as a strategy for **close together numbers** (e.g. 121—118), and also for numbers that are "nearly" multiples of 10, 100, 1000 or £s, which make it easier to count on (e.g. 102-89, 131—79, or calculating change from £1 etc.). Start at the smaller number and count on in **tens** first, then count on in units to find the rest of the difference:

Because counting on in tens is the way we use a 100 square.





Year 4 Subtract with up to 4-digit numbers

Partitioned column subtraction with „exchanging" (decomposition):

2754 – 1562 = 1192

2000 + 600 + 100 + 50 + 4

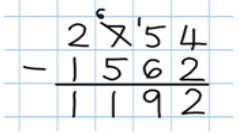
– 1000 + 500 + 60 + 2

1000 + 100 + 90 + 2

As introduced in Y3, but moving towards more complex numbers and values. Use **place value counters** to reinforce exchanging".

Subtracting money: partition into £1 + 30 + 5 for example.

Compact column subtraction (see video)



To introduce the compact method, ask children to perform a subtraction calculation with the familiar partitioned column subtraction then display the compact version for the calculation. Discuss how they have done. Ask pupils to consider how it relates to the method they know, what is similar and what is different, to develop an understanding of it (shown on video).

Give plenty of opportunities to apply this to money and measures.

Always encourage children to consider the best method for the numbers involved— mental, counting on, counting back or writ- ten method (see video).

Mental strategies

A variety of mental strategies must be taught and practised, including counting on to find the difference where numbers are closer together, or where it is easier to count on (see video below).

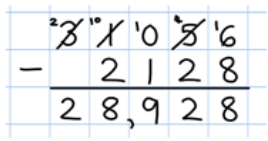
Approximate, Calculate, Check it make!

Year 5 Subtract with at least 4-digit numbers

including money, measures, decimals.

Compact column subtraction

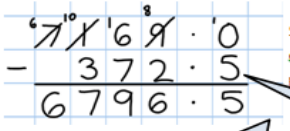
(with exchanging").



Children who are still not secure with number facts and place value will need to remain on the partitioned column method until ready for the compact method.

Subtracting with larger integers.

See moving to the compact method video.

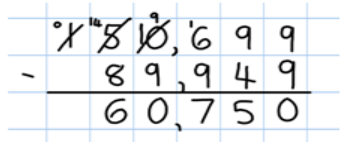


Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.

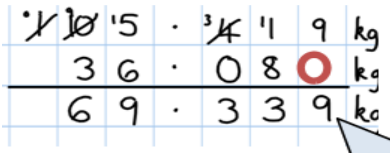
Create lots of opportunities for subtracting and finding differences with money and measures.

Add a zero" in any empty decimal places to aid understanding of what to subtract in that column.

Year 6 Subtracting with increasingly large and more complex numbers and decimal values.



Using the compact column method to subtract more complex integers



Using the compact column method to subtract money and measures, including decimals with different numbers of decimal places.

Empty decimal places can be filled with **zero** to show the place value in each column.

Pupils should be able to apply their knowledge of a range of mental strategies, mental recall skills, and informal and formal written methods when selecting **the most appropriate method** to work out subtraction problems.

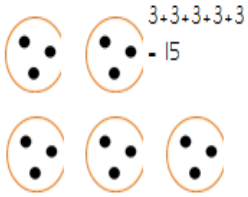
Progression in Written Multiplication

Year 1 Multiply with concrete objects, arrays and pictorial representations.

How many legs will 3 teddies have?

2 + 2 + 2 = 6

There are 3 sweets in one bag.
How many sweets are in 5 bags altogether?



3+3+3+3+3 = 15

Give children experience of counting equal group of objects in 2s, 5s and 10s.

Present practical problem solving activities involving counting equal sets or groups, as above.

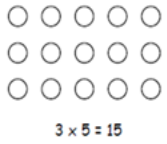
Year 2 Multiply using arrays and repeated addition (using at least 2s, 5s and 10s)

Use repeated addition on a number line:

Starting from zero, make equal jumps up on a number line to work out multiplication facts and write multiplication statements using x and = signs.

4 X 5 = 20

Use arrays:



5 x 3 = 15

5 x 3 = 3 + 3 + 3 + 3 + 3 = 15

3 x 5 = 5 + 5 + 5 = 15

3 x 5 = 15

Use arrays to help teach children to understand the commutative law of multiplication, and give examples such as 3 x __ = 6.

Use practical apparatus:

5 x 3 = 5 + 5 + 5

Use mental recall:

Children should begin to **recall multiplication facts for 2, 5 and 10** times tables through practice in counting and understanding of the operation.

Year 3 Multiply 2-digits by a single digit number

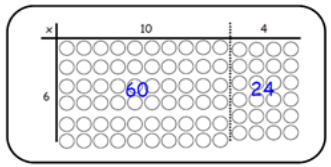
Introduce the grid method for multiplying 2-digit by single-digits:

Eg. 23 x 8 = 184

X	20	3
8	160	24

160 + 24 = 184

Link the layout of the grid to an array initially:



Introduce the grid method with children physically making an array to represent the calculation (e.g. make 8 lots of 23 with 10s and 1s place value counters), then translate this to grid method format (see video clip).

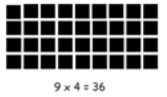
To do this, children must be able to:

Partition numbers into tens and units

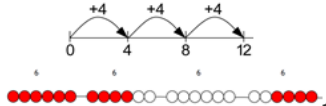
Multiply multiples of ten by a single digit (e.g. 20 x 4) using their knowledge of multiplication facts and place value

Recall and work out multiplication facts in the **2, 3, 4, 5, 8 and 10** times tables.

Work out multiplication facts not known by repeated addition or other taught mental strategies (e.g. by commutative law, working out near multiples and adjusting, using doubling etc.) Strategies to support this are repeated addition using a number line, bead bars and arrays:



9 x 4 = 36



Year 4 Multiply 2 and 3-digits by a single digit, using all multiplication tables up to 12 x 12

Developing the grid method:

Eg. 136 x 5 = 680

X	100	30	6
5	500	150	30

500
150
+ 30
680

Encourage column addition to add accurately.

Move onto **short multiplication** (see Y5) if and when children are confident and accurate multiplying 2 and 3-digit numbers by a single digit this way, **and** are already confident in "carrying" for written addition.

Children should be able to:

Approximate before they calculate, and make this a regular part of their calculating, going back to the approximation to check the reasonableness of their answer. e.g:

346 x 9 is approximately 350 x 10 = 3500!!

Record an approximation to check the final answer against.

Multiply multiples of ten and one hundred by a single-digit, using their multiplication table knowledge.

Recall all times tables up to 12 x 12

Approximate, Calculate, Check it mate!

Multiplication

Year 5 Multiply up to 4-digits by 1 or 2 digits.

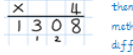
Introducing column multiplication

Introduce by comparing a grid method calculation to a short multiplication method, to see how the steps are related, but notice how there are less steps involved in the column method (see video).

Children need to be taught to approximate first, e.g. for **72 x 38, they will use rounding:** 72 x 38 is approximately 70 x 40 = 2800, and use the approximation to check the reasonableness of their answer against.

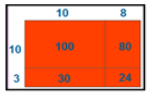
Short multiplication for multiplying by a single digit

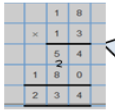
x	300	20	7
4	1200	80	28



Pupils could be asked to work out a given calculation using the grid, and then compare it to your column method. What are the similarities and differences? Unpick the steps and show how it reduces the steps.

Introduce long multiplication for multiplying by 2 digits



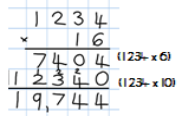


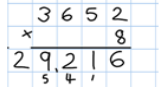
18 x 3 on the 1st row (8 x 3 = 24, carrying the 2 for twenty, then 1 x 3).

18 x 10 on the 2nd row. Put a zero in units first, then say 8 x 1, and 1 x 1.

Moving towards more complex numbers:

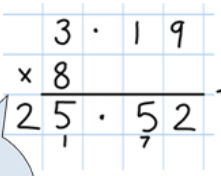
The grid could be used to introduce long multiplication, as the relationship can be seen in the answers in each row.





Approximate, Calculate, Check it mate!

Year 6 Short and long multiplication as in Y5, and multiply decimals with up to 2d.p by a single digit.



Line up the decimal points in the question and the answer.

Remind children that the single digit belongs in the units column.

This works well for multiplying money (£.p) and other measures.

Children will be able to:

Use rounding and place value to make approximations before calculating and use these to check answers against.

Use **short multiplication** (see Y5) to multiply numbers with **more than 4-digits by a single digit** to multiply money and measures, and to **multiply decimals with up to 2d.p. by a single digit.**

Use **long multiplication** (see Y5) to multiply numbers with **at least 4 digits by a 2-digit number.**

Progression in Written Division

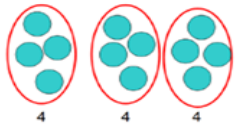
Year 1 Group and share small quantities

Using objects, diagrams and pictorial representations to solve problems involving both grouping and sharing.

How many groups of 4 can be made with 12 stars? – 3

Grouping:

Sharing:



12 shared between 3 is 4

Example division problem in a familiar context:

There are 6 pupils on this table and there are 18 pieces of fruit to share between us. If we share them equally, how many will we each get?

Can they work it out and give a division statement...?

"18 shared between 6 people gives you 3 each."

Pupils should:

use lots of practical apparatus, arrays and picture representations

Be taught to understand the difference between 'grouping' objects (How many groups of 2 can you make?) and 'sharing' (Share these sweets between 2 people)

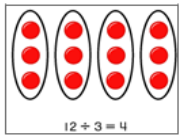
Be able to count in multiples of 2s, 5s and 10s.

Find **half** of a group of objects by sharing into 2 equal groups.

Year 2 Group and share, using the ÷ and – sign

Use objects, arrays, diagrams and pictorial representations, and grouping on a number line.

Arrays:

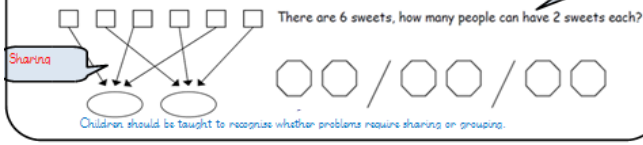


This represents **12 ÷ 3, posed as** how many groups of 3 are in 12?

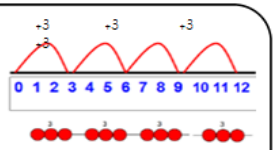
Pupils should also show that the same array can represent **12 ÷ 4 = 3** if grouped horizontally.

Know and understand sharing and grouping:

6 sweets shared between 2 people, how many do they each get?



Grouping using a number line:



Group from zero in equal jumps of the divisor to find out "how many groups of _ in _ ?". Pupils could and using a bead string or practical apparatus to work out problems like A CD costs £3. How many CDs can I buy with £12? **This is an important method to develop understanding of division as grouping.**

12 ÷ 3 = 4

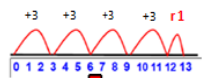
Pose **12 ÷ 3** as **How many groups of 3 are in 12?**

Division

Year 3 Divide 2-digit numbers by a single digit (where there is no remainder in the final answer)

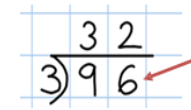
Grouping on a number line:

13 ÷ 3 = 4 r 1



STEP 1: Children continue to work out unknown division facts by grouping on a number line from zero. They are also now taught the concept of **remainders**, as in the example. This should be introduced practically and with arrays, as well as being translated to a number line. Children should work towards calculating some basic division facts with remainders mentally for the 2s, 3s, 4s, 5s, 8s and 10s, ready for 'carrying' remainders across within the short division method.

Short division: Limit numbers to NO remainders in the answer, OR carried (each digit must be a multiple of the divisor).



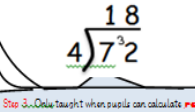
STEP 2: Once children are secure with division as grouping and demonstrate this using number lines, arrays etc., **short division** for larger 2-digit numbers should be introduced, initially with carefully selected examples requiring no calculating of remainders at all. Start by introducing the layout of short division by comparing it to an array.

Remind children of correct place value, that 96 is equal to 90 and 6, but in short division, pose:

How many 3's in 90 – 3, and record it above the **9 tens**.

How many 3's in 6? – 2, and record it above the **6 units**.

Short division: Limit numbers to NO remainders in the final answer, but with remainders occurring within the



STEP 3: Once children demonstrate a full understanding of remainders, and also the short division method taught, they can be taught how to use the method when remainders occur within the calculation (e.g. 96÷4), and be taught to carry the remainder onto the next digit. **If needed, children should use the number line to work out individual division facts that occur which they are not yet able to recall mentally.**

Real life contexts need to be used routinely to help pupils gain a full understanding, and the ability to recognise the place of division and how to apply it to problems.

Step 3...Only taught when pupils can calculate **remainders**.

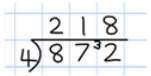
Division

Year 4 Divide up to 3-digit numbers by a single digit (without remainders initially)

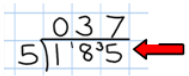
Continue to develop short division:

Short division should only be taught once children have secured the skill of calculating 'remainders'.

STEP 1: Pupils must be secure with the process of **exact** division for dividing 2-digit numbers by a single digit **(those that do not result in a final remainder** —see steps in Y3), but must understand how to calculate remainders, using this to 'carry' remainders within the calculation process (see example).



STEP 2: Pupils move on to dividing numbers with up to **3-digits** by a single digit, however problems and calculations provided should **not result in a final answer with remainder** at this stage. Children who exceed this expectation may progress to Y5 level.



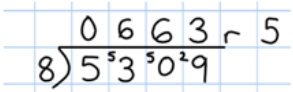
When the answer for the **first column** is zero (1 ÷ 5, as in example), children could initially write a zero above to acknowledge its place, and must always 'carry' the number (1) over to the next digit as a remainder.

Include money and measure contexts when confident.

Real life contexts need to be used routinely to help pupils gain a full understanding, and the ability to recognise the place of division and how to apply it to problems.

Year 5 Divide up to 4 digits by a single digit, including those with remainders.

Short division, including remainder answers:



Short division with remainders: Now that pupils are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where **pupils consider the meaning of the remainder and how to express it**, ie. as a fraction, a decimal, or as a rounded number or value, depending upon the context of the problem.

The answer to 5309 ÷ 8 could be expressed as 663 and five eighths, 663 r 5, as a decimal, or rounded as appropriate to the problem involved.

See Y6 for how to continue the short division to give a decimal answer for children who are confident.

Include money and measure contexts.

Approximate, Calculate, Check it make!

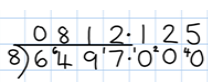
If children are confident and accurate:

Introduce **long division** for pupils who are ready to divide any number by a 2-digit number (e.g. 2678 ÷ 19). This is a Year 6 expectation...see

Division

Year 6 Divide at least 4 digits by both single-digit and 2-digit numbers (including decimal numbers and quantities)

Short division, for dividing by a single digit: e.g. 6497 ÷ 8



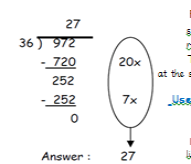
Short division with remainders: Pupils should continue to use this method, but with numbers to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers. Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.

Calculating a decimal remainder: In this example, rather than expressing the remainder as **r 1**, a decimal point is added after the units because there is still a remainder, and the one remainder is carried onto zeros after the decimal point (to show there was no decimal value in the original number).

Keep dividing to an appropriate degree of accuracy for the problem being solved.

Must be aligned in place value for subtracting.

Introduce long division by chunking for dividing by 2 digits.



Find out How many 36s are in 972? by subtracting 'chunks' of 36, until zero is reached (or until there is a remainder).

Teach pupils to write a **useful list**, first at the side that will help them decide what chunks to use, e.g.:

Useful list:
1x = 36
10x = 360
100x = 3600

Introduce the method in a simple way by limiting the choice of chunks to Can we use 10 lots? Can we use 100 lots? As children become confident with the process, encourage more efficient chunks to get to the answer more quickly (e.g. 20x, 5x), and expand on their 'useful' lists.

Where **remainders** occur, pupils should express them as fractions, decimals or use rounding, depending upon the problem.

Approximate, Calculate, Check it make!